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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2023

MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE** unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. Two blank pages (pages 27–28) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
14	12	12	13	10	15	10	12	10	17	17	8	150

SECTION A

QUESTION 1

(a) Solve for x in each case below:

(1) $2 = 3^x$ (leave answer in logarithmic form)

(1)

(2) $\sqrt{x+7} - 1 = x$

(4)

(b) Solve for x and y simultaneously if:

$y - 2x = 7$ and $x^2 - x - y - 3 = 0$

(5)

(c) For what values of x is $3x^2 \leq x$?

(4)
[14]

QUESTION 2

If $f(x)=(x+2)(x-4)(x-4)$ then:

(a) Determine the coordinates for the *y*-intercept of the graph of *f*.

(1)

(b) Determine $f'(x)$.

(3)

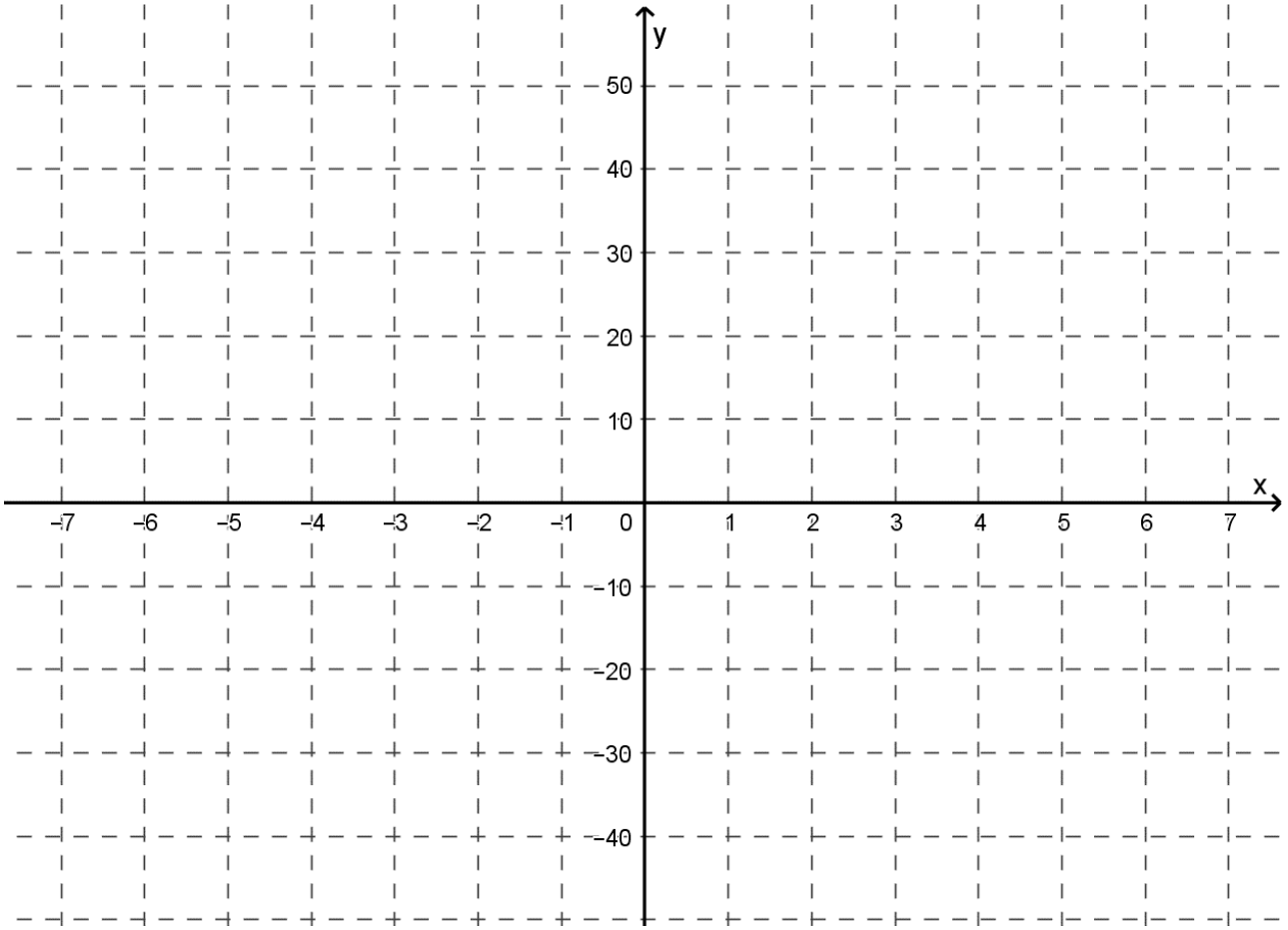
(c) For what values of *x* will $f'(x)=0$?

(2)

(d) On the set of axes provided sketch the graph of *f.*

Label the following:

- Both *x* and *y* intercepts
- Turning points or stationary points
- Point of inflection



(4)

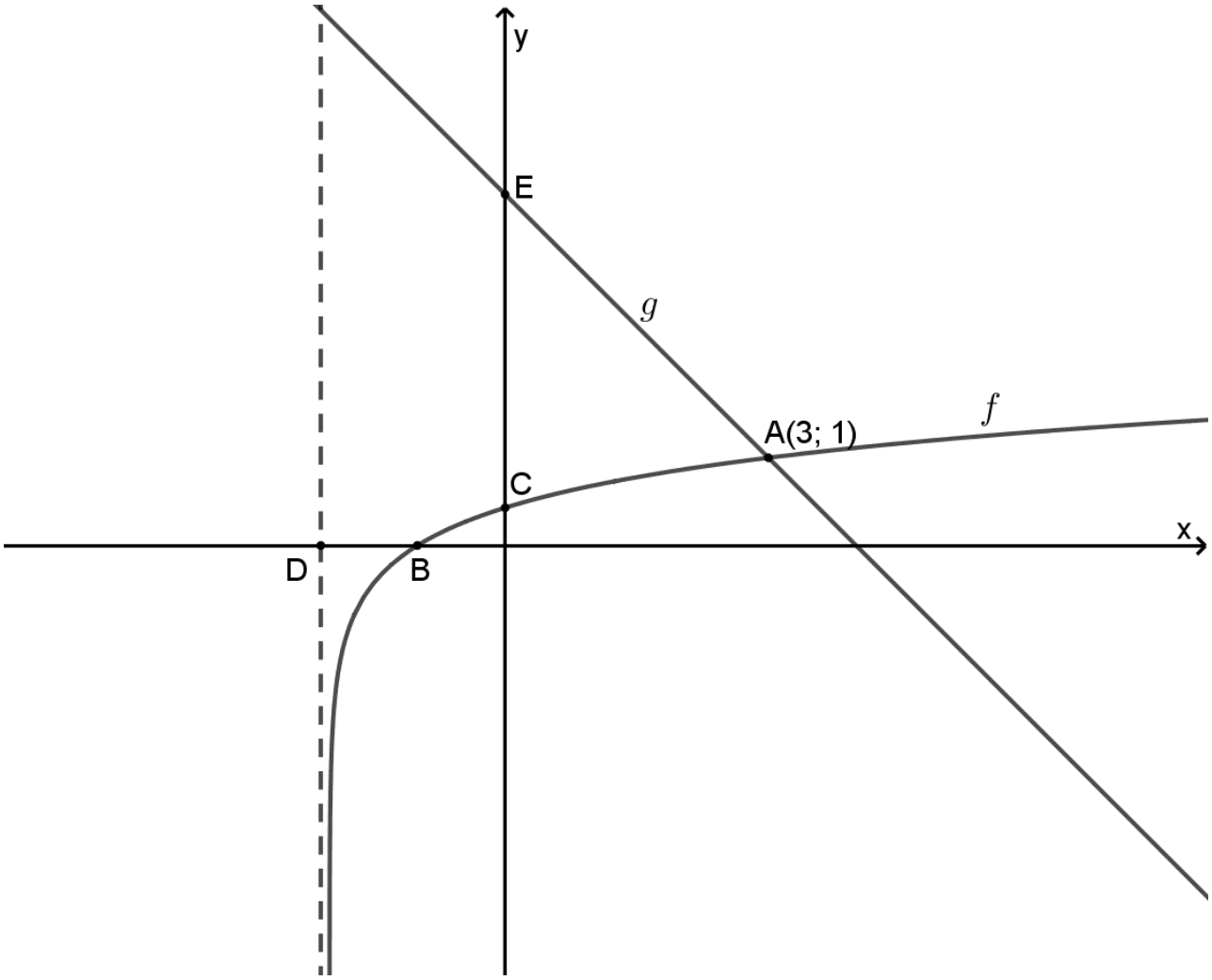
(e) For what values of *x* is $f(x) \geq 0$?

(2)
[12]

QUESTION 3

The graphs of *f* and *g* have been sketched on the set of axes below.

- $f(x) = \log_m(x+2)$
- $g(x) = -x + 4$
- B and C are the x- and y-intercept of the graph of *f* respectively.
- $A(3;1)$ lies on both *f* and *g*.
- The dotted line through D with equation $x = -2$ is the asymptote for *f.*



(a) $f(x) = \log_m(x+2)$; determine the value of *m*.

(2)

(b) Determine the coordinates of B and C if $m = 5$.

(4)

(c) For what value(s) of x is $f(x) \leq g(x)$?

(2)

(d) On the same set of axes sketch the graph of f^{-1}, the inverse of f.
Label the intercepts, asymptote and the point where $f^{-1}(x) = g(x)$.

(4)
[12]

QUESTION 4

(a) Find $f'(x)$ and $g'(x)$, respectively, for each of the following:

(1) $f(x) = \sqrt[5]{x^2} - \frac{3}{x^3}$

(4)

(2) $g(x) = \frac{x^2 - 2x + 1}{4x - 4}$

(3)

(b) The following information is given for a function *f*.

- $f(4) = 5$
- $f'(4) = 7$

Determine the equation of the tangent to *f* at $x = 4$ in the form of $y = mx + c$.

(3)

(c) Using first principles, find $h'(x)$ if $h(x) = 5x - 8$.

(3)
[13]

QUESTION 5

Consider the arithmetic sequence given:

$$22; 25; 28; 31; \ldots ; 262$$

(a) Determine the number of terms in the sequence.

(3)

(b) Represent the sum of all the terms using sigma notation.

(2)

(c) If the pattern continues, determine the smallest number of terms that must be added to the sequence above so that the sum of all the terms will be more than 15 000.

(5)
[10]

QUESTION 6

(a) You invest into two different investment schemes for a period of 5 (five) years.

Investment 1:

You place R3 500 into an investment at the end of each month.
Interest on this investment is 15% per annum compounded monthly.

Investment 2:

You make a deposit of R24 000 immediately and then only make a second deposit of R7 000 into the same account at the end of the 3rd (third) year.
Interest on this investment is 20% per annum compounded quarterly.

If you combine the value of your two investments at the end of the five-year period, then what is the value of this lump sum?

Give your answer correct to two decimal places.

(7)

(b) To start your ice cream business you take out a loan for R250 000.
The bank offers an interest rate of 10% per annum compounded monthly.

(1) How much should you pay each month if you would like to pay the loan off at the end of ten years, if the first payment is made at the end of the first month? Give your answer correct to two decimal places.

(3)

(2) If you decide to pay R6 000 per month, only starting at the end of the second month, then how many payments will be needed to pay off the loan? (Round up to the nearest number of payments.)

(5)
[15]

76 marks

SECTION B

QUESTION 7

(a) If $x^2 - 16x + 73 = a(x - m)^2 + p$ then determine the value of m and p.

(3)

(b) If $2^x = k$, determine the following in terms of k.

(1) 2^{2x}

(1)

(2) $2^{\frac{x}{3}}$

(1)

(3) 2^{-x+1}

(1)

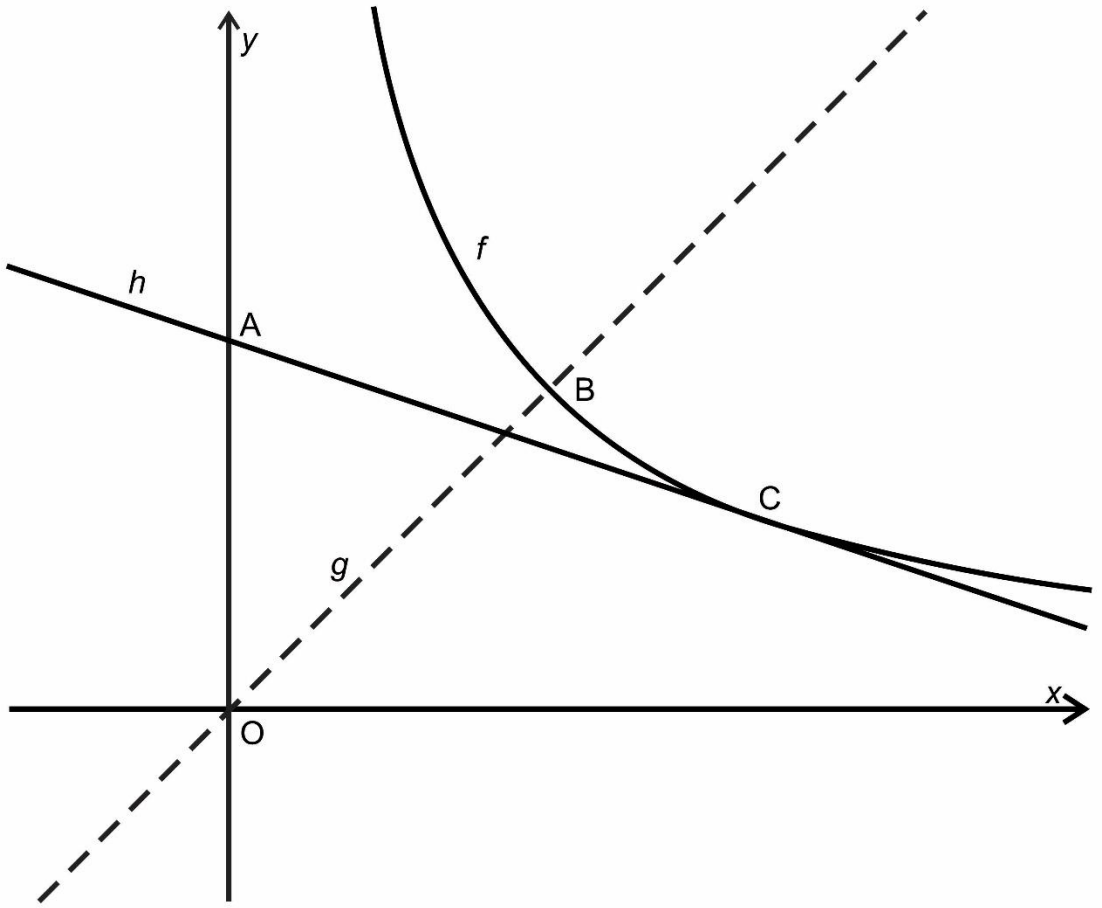
(c) Solve for p if $\frac{3^{\log_p 2}}{3^{\log_p 8}} = 9$

(4)
[10]

QUESTION 8

In the diagram below:

- $f(x)=\frac{12}{x}$ for $x>0$.
- g is the axis of symmetry for f.
- B is the point where f intersects g.
- h is a tangent to f at C.
- A is the y-intercept of h.



(a) Write down the equation of g.

(1)

(b) Determine the coordinates of B. (Leave answers in surd form.)

(3)

(c) Determine the area of ΔACO if $h(x) = -\frac{1}{3}x + c$.

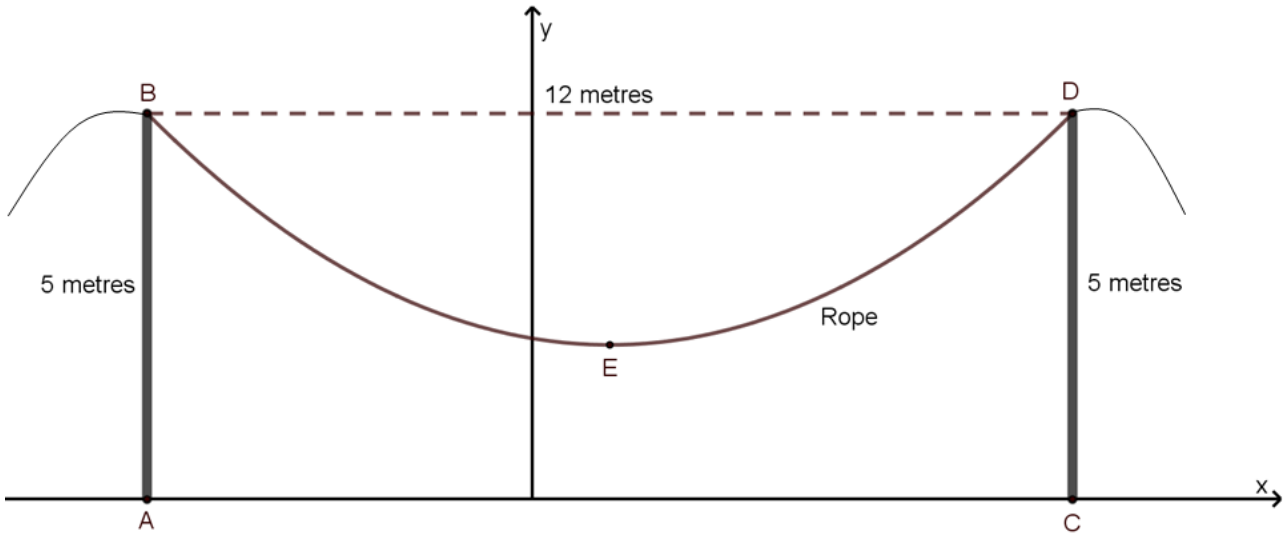
(8)
[12]

QUESTION 9

In the Cartesian plane below, a piece of rope is hanging over the top of two vertical poles AB and CD.

Key information:

- The poles remain vertical at all times and are 5 metres in height.
- The horizontal distance between B and D is 12 metres.
- Point $\mathrm{E}(1;2)$ represents the lowest point of the rope.



(a) If the shape of the rope between the points B and D is modelled by the equation $y = a(x-p)^2 + q;$ then determine the value of a.

(4)

(b) If pole CD is shifted to the right, then what should the horizontal distance between the two vertical poles AB and CD become, so that the value of a is $\frac{1}{100}$ and the lowest point of the rope is 4 metres above the ground?

(6)
[10]

QUESTION 10

(a) A pool is being filled up by a hosepipe. The rate of flow increases by 10% each hour and the table below illustrates this process.

Time	hour 1	hour 2	hour 3	hour 4
Rate of flow	1 litre/minute	1,1 litre/minute	1,21 litre/minute	1,331 litre/minute

(1) Calculate the rate of flow during hour 24.
(Assume the geometric pattern above continues.)

(2)

(2) If the hosepipe above is filling up a pool, then how many litres of water would be in the pool at the end of 24 hours? (Assuming the pool was empty at the start of the process.)

(3)

(b) For a quadratic sequence the following information is given:

$T_2 - T_1 = 4 ; T_3 - T_2 = 7$ and $T_{10} = 0$

Determine the *nth* term of the quadratic sequence.

(5)

(c) The first term of a converging geometric series is equal to the common ratio; if $S_\infty = \frac{1}{5}$, then calculate the value of the first term in the series.

(3)

(d) Consider the geometric sequence given:

$x; y; x + y; \ldots$ where $x > 0$ and $y > 0$

Determine the numerical value for $\frac{x}{y}$.

(4)
[17]

QUESTION 11

(a) After some research you decide that the best option for your online security is to have two unique passwords before opening important documents.

(1) The first ten-digit password consists of two parts.

(i) The first part is made up of six numbers. The numbers that can be used are the numbers from 1 to 9. How many unique six-digit codes can be created if ***repetition is allowed***?

(2)

(ii) The second part is a four-digit code compiled from the digits 1 to 9, ***repetition is not allowed***. How many **even** ten-digit passwords can be created by combining these two parts?

(3)

(2) The second nine-digit password has to be a number greater than 600 000 000 with the last digit being divisible by 3. How many nine-digit passwords are possible if you can use the digits from 1 to 9, ***no repetition is allowed***?

(4)

(b) You have two regular six-sided dice; one is green and one is red.
You throw both of the dice at the same time and record the values.
$g =$ the value on the green dice.
$r =$ the value on the red dice.
Once you have thrown the two dice you place the values into the expression for $f(x)$.

If $f(x) = (x + g)(x - r)$ then answer the questions that follow:

(1) What is the probability that the y intercept for the graph of f will be less than –29?

(4)

(2) What is the probability that the roots of $f(x) = 0$ will be real, rational and unequal?

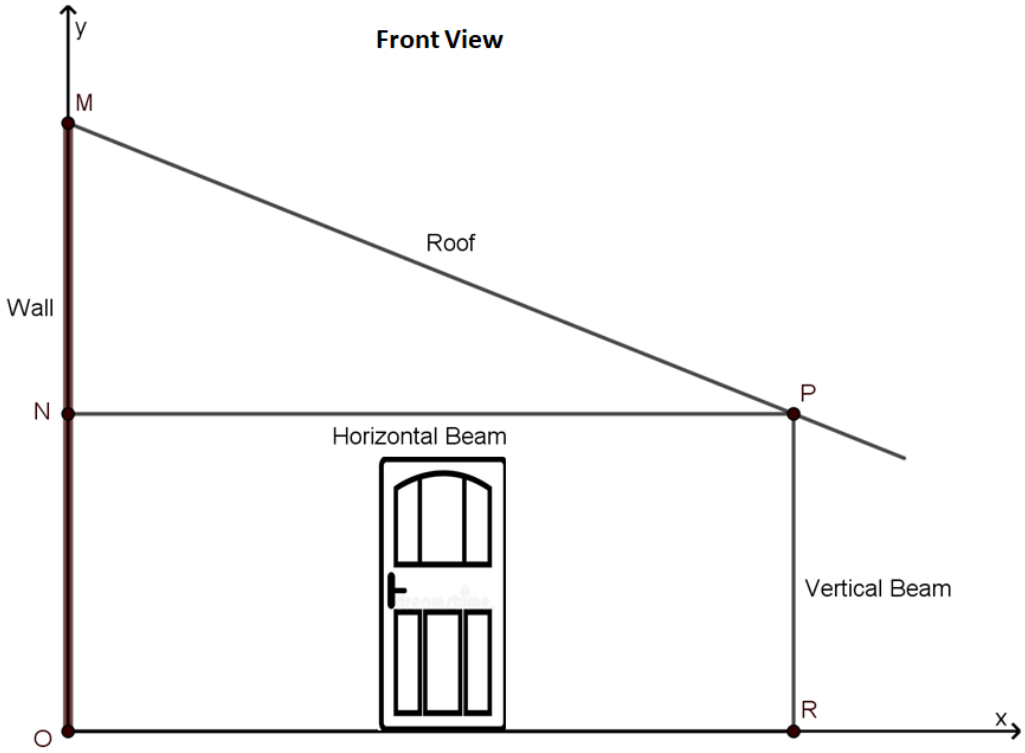
(1)

(3) If $h(x) = -1(x+g)^2 + 4 - r$ then what is the probability that if you throw the two dice and place the values into $h(x)$ that the roots of $h(x) = 0$ will be non-real?

(3)
[17]

QUESTION 12

Below is the front view of a rectangular storage facility built onto a wall represented by MO in the diagram below.

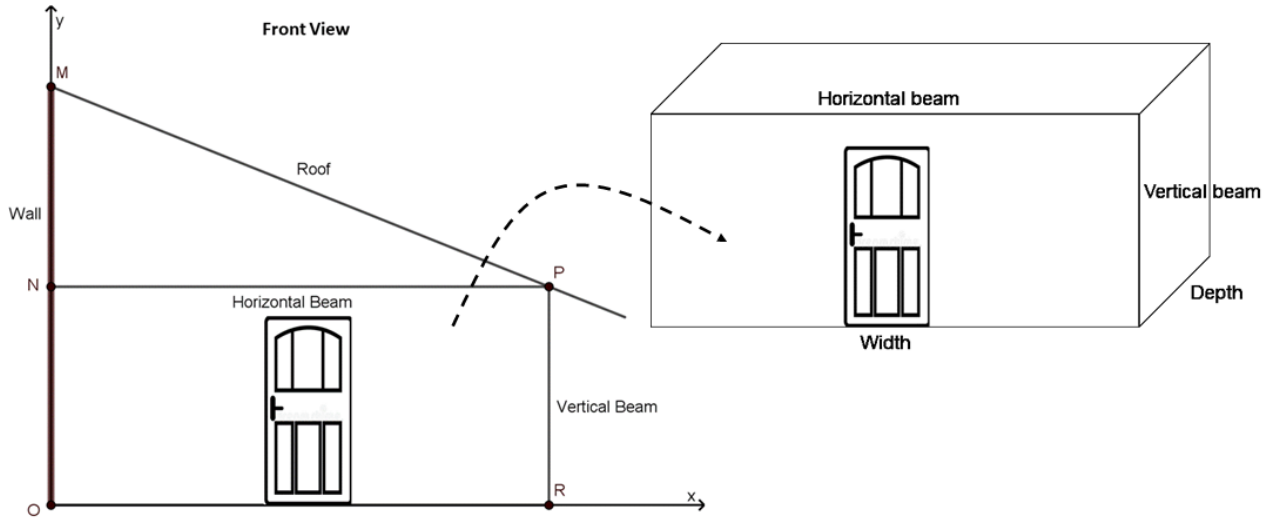


- The roof is attached to the wall at M; with MO = 6 metres.
- The gradient of the roof MN:NP must have a ratio of 2:5.
- PR is a vertical beam.
- NP is a horizontal beam.
- OR represents the ground.

(a) Determine the equation of the line MP that represents the roof.

(2)

(b) Below is the rectangular storage facility under the roof MP, shown in three-dimensional form.



If the ***depth*** of the rectangular storage facility is two metres shorter than the width, then determine the length of the vertical beam, so that the rectangular storage facility will have a maximum volume when placed under the roof MP. **(The roof must remain fixed at M and the ratio of MN:NP must remain 2:5)**

(6)
[8]

74 marks

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

